Technical Comments

Comments on "Energy Considerations for Attitude Stability of Dual-Spin Spacecraft"

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The dual-spin concept of satellite design has evoked much interest in recent years, and the application of energy methods to the analysis of such systems has provided results which have been useful to designers in spite of their approximate nature. In a recent Technical Note,[1] B. T. Fang employs energy methods to study the behavior of a dual-spin system that supposedly admits the possibility of energy dissipation on both of the rotating sections. A stability criterion is obtained by heuristic arguments, and behavior is discussed. As will be shown, however, application of the aforementioned stability criterion leads to a contradiction which indicates the necessity of re-examining the assumptions underlying Fang's development.

To discuss meaningfully the stability of motion of a physical system, one must carefully describe the particular motion whose stability is of interest. For the system described in Ref. 1, consider the motion consisting of pure spin in which the rotor axis is aligned with the system angular momentum vector. Further, let this motion be called asymptotically stable if and only if the system responds to a slight initial disturbance by returning to a similar state of pure spin for a slightly displaced angular momentum vector. Using this definition together with the heuristic arguments advanced by Fang, it follows that asymptotic stability will obtain if and only if the magnitude of a quantity μ defined as

$$\mu = I_3{}^S\omega_3{}^S(t_0)/h(1 - I_3{}^A/I_1) \tag{1}$$

is greater than or equal to one. (The symbols used are defined in Ref. 1.) An important restriction in the class of systems to which this stability criterion is applicable is illustrated by the following example.

Consider a physical system for which

$$\begin{gathered} I_1 = I_2 = I_T,\ I_3{}^A = I_P,\ I_3{}^S = J_P \\ \omega_3{}^A(t_0) = \Omega,\ \omega_3{}^S(t_0) = \sigma \end{gathered} \tag{2}$$

The magnitude h of the system angular momentum vector can be expressed as

$$h = \{I_T[\omega_1(t_0)^2 + \omega_2(t_0)^2] + (I_P\Omega + J_P\sigma)^2\}^{1/2} \tag{3}$$

and the value of μ becomes

$$\mu_1 = J_P\sigma/[h(1 - I_P/I_T)] \tag{4}$$

If a second physical system is now considered for which

$$\begin{gathered} I_1 = I_2 = I_T,\ I_3{}^A = J_P,\ I_3{}^S = I_P \\ \omega_3{}^A(t_0) = \sigma,\ \omega_3{}^S(t_0) = \Omega \end{gathered} \tag{5}$$

then the value of h is again given by (3) and μ becomes

$$\mu_2 = I_P\Omega/[h(1 - J_P/I_T)] \tag{6}$$

It is clear that the values of μ_1 and μ_2 will, in general, be different. For example, if $\Omega = 0$ and $I_P = I_T$,† then

$$\mu_1 = \infty \tag{7}$$

while

$$\mu_2 = 0 \tag{8}$$

By applying the stability criterion given previously, one concludes that the first system is unstable whereas the second is asymptotically stable. However, if one assumes, as Fang does, that dissipating mechanisms of a relatively unrestricted description are allowed on both bodies in each system, then the two systems may be made physically equivalent, in which case they should exhibit the same behavior. The two parallel analytical developments differ only in the labeling of the two symmetrical sections of each system, i.e., in the arbitrary designation of one of the bodies as A and the other as S. The apparent contradiction which involves the prediction of two completely different kinds of behavior for the same physical system can be resolved by examining assumption 1 and Eq. (1) in Fang's Note.

According to assumption 1, dissipation of a relatively general description can occur in either body. Equation (1), however, requires that $\omega_3{}^S$ be a constant. For most systems of practical interest, these requirements are mutually exclusive since the spin rate of a dissipative rotor will not, in general, remain constant even if the rotor is nominally symmetrical. Assumption 1 and Eq. (1) are therefore consistent only for a very special kind of internal dissipation mechanism—one which does not affect the value of $\omega_3{}^S$. Although it may be possible to conceive of such damping mechanisms theoretically, they are of little practical interest and represent a small improvement over the assumption of a completely rigid rotor.

The energy dissipating mechanisms in the (possibly unsymmetrical) body A are not similarly restricted. Therefore it must be concluded that, in addition to possible differences in inertia properties, bodies A and S differ significantly in the character of the dissipation which each can permit. This remains true even if A is presumed to be a nominally symmetrical body. Thus the apparent contradiction described previously is resolved by the observation that even for a nominally symmetrical system, the implicit assumption in Fang's development that the mechanisms for energy dissipation on one of the rotating sections are basically different from those on the other section excludes the possibility of arbitrarily designating one body as A and the other as S.

In summary, it must be emphasized that the stability criterion developed by Fang can be considered valid only when the energy dissipation on the rotor S results from a severely restricted class of dissipating mechanisms as described previously. The statement of assumption 1 should be altered to reflect this fact. That stability criteria for dual-spin systems with energy dissipation on both bodies will, in general, differ substantially from that proposed by Fang is demonstrated in Ref. 2.

References

[1] Fang, B. T., "Energy Considerations for Attitude Stability of Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1241–1243.

[2] Mingori, D. L., "Effects of Energy Dissipation on the Attitude Stability of Dual Spin Satellites," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 20–27.

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† This selection of inertia parameters is permissible because the stated assumption that $I_2 > I_3{}^A + I_3{}^S$ is not used in Fang's development.